Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Student number\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Assignment 3**

Consider a cantilever in *xy*-plane loaded by distributed force  and point force . Determine the displacement and rotation of the free end. Young’s modulus  and Poisson’s ratio are constants. Cross-section is a rectangle of side length *t*. Assume that the neutral axis coincides with the *x*-axis of the material coordinate system. Use the bar and bending modes of the beam model.

*x,X*

*L*

*y,Y*



*f*

 *45*

**Solution template**

Assuming that the material coordinate system is chosen so that the bending and stretching modes decouple, the two modes can be taken into account as if they were separate elements. Therefore, one may use the virtual work expressions for the beam *xy*-plane bending and bar modes of the formulae collection

, ,

, .

The nodal displacements and rotations of the material coordinate systems need to be expressed in terms of those of the structural coordinate system. By using the figure

, , ,

, , .

The cross-section properties and the distributed force (per unit length) components in the material coordinate system are

, , , .

When the relationships are used in the element contribution of the beam bending mode the generic expressions simplify to

.

The bar mode expression takes the form

.

Virtual work expression of the point force

.

Virtual work expression is the sum of the element/mode contributions

.

Principle of virtual work and the fundamental lemma of variational calculus imply the linear equations system

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The first equation is not connected to the second and third. Therefore, the solution can be found without inverting the 3-by-3 matrix (the bending mode equations are connected so a 2-by-2 matrix needs to be inverted)

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